7.4 Solution Curves of Linear Systems (continued)

$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \vec{x}$

$\lambda = 1, 1$

$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ only $\quad$ need a generalized eigenvector

let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$(A - \lambda I)\vec{v}_2 = \vec{v}_1 \qquad \left[\begin{array}{cc|c} 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right] \qquad \vec{v}_2 = \begin{bmatrix} r \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (r=0)$

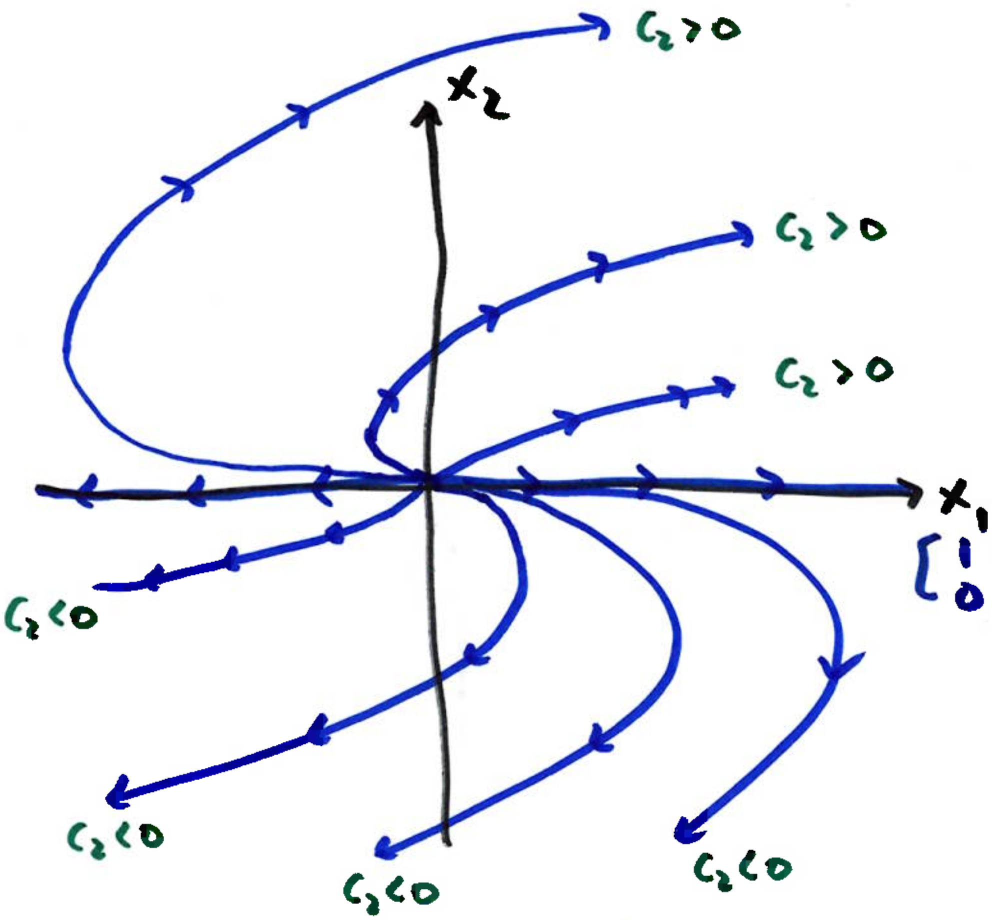
Solution: $\vec{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^t \left(t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$

as $t \to \infty$, both e^t and te^t are important

both $e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ follow the same vector: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\rightarrow$ ordinary eigenvector

the generalized one: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ affects orientation but is not really visible

Only one asymptote : $\begin{bmatrix}1\\0\end{bmatrix}$ (ordinary eigenvector)



$\begin{bmatrix}1\\0\end{bmatrix}$, $\lambda>0$
(away from origin)

Origin is an improper nodal source
(defective matrix)

if ~~def~~ matrix is complete then proper

if $c_2>0$, $c_2e^t\begin{bmatrix}0\\1\end{bmatrix}$ gives the solutions an "up" direction as t increases

$c_2<0$, "down" direction as t increases

$$\vec{x}' = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \vec{x}$$

$\lambda = 0,\ 1$

$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

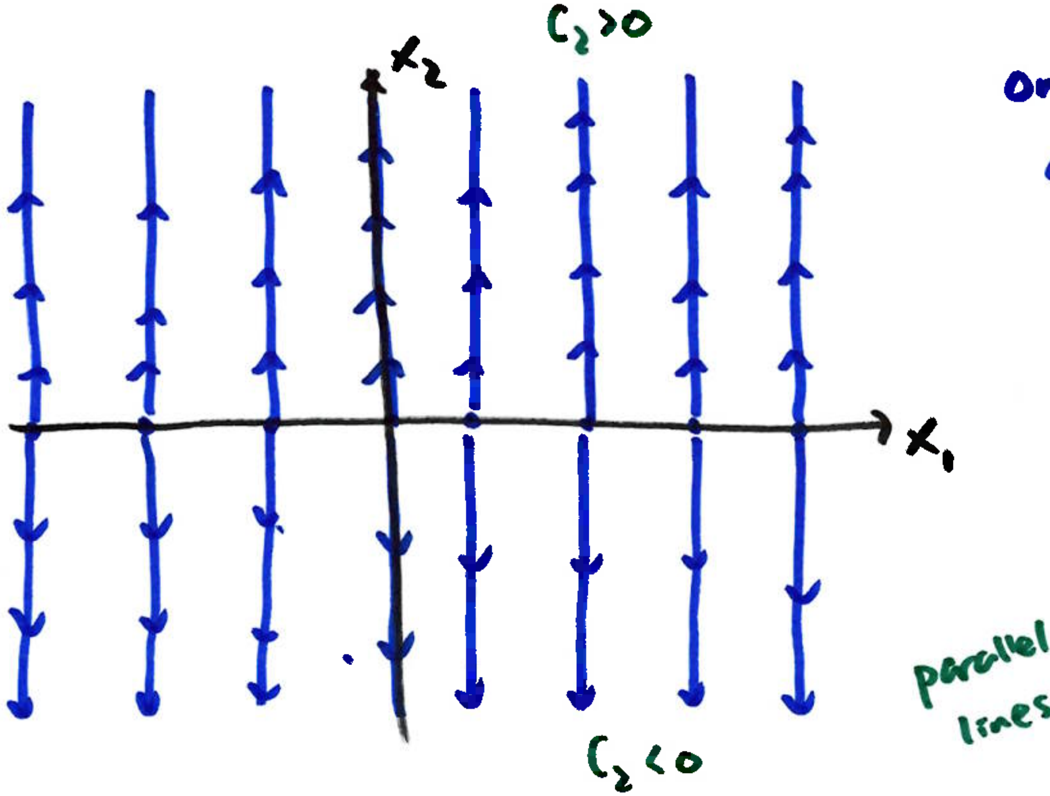
$$\vec{x}(t) = c_1 e^{0 \cdot t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$x_1(t) = c_1$

$x_2(t) = c_2 e^{t}$

solutions are up/down

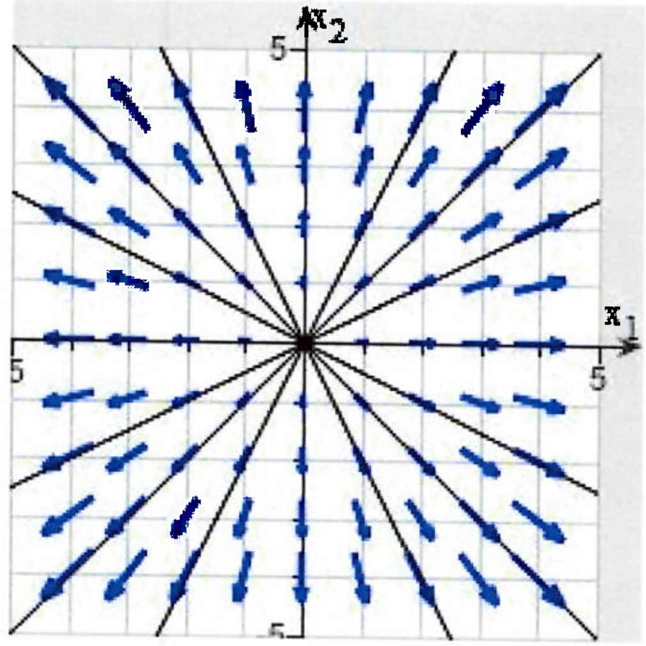


On the x_1-axis all points are $\begin{bmatrix} k \\ 0 \end{bmatrix}$

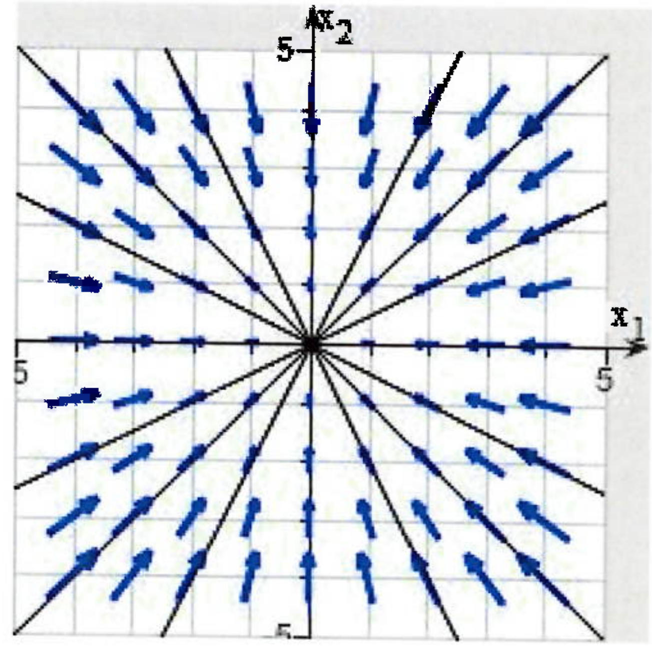
$$\vec{x}' = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ no tangent vectors

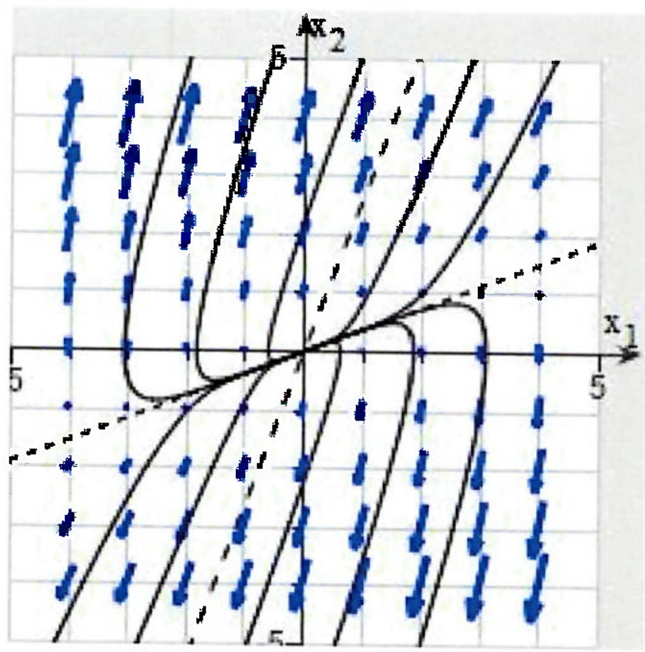
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

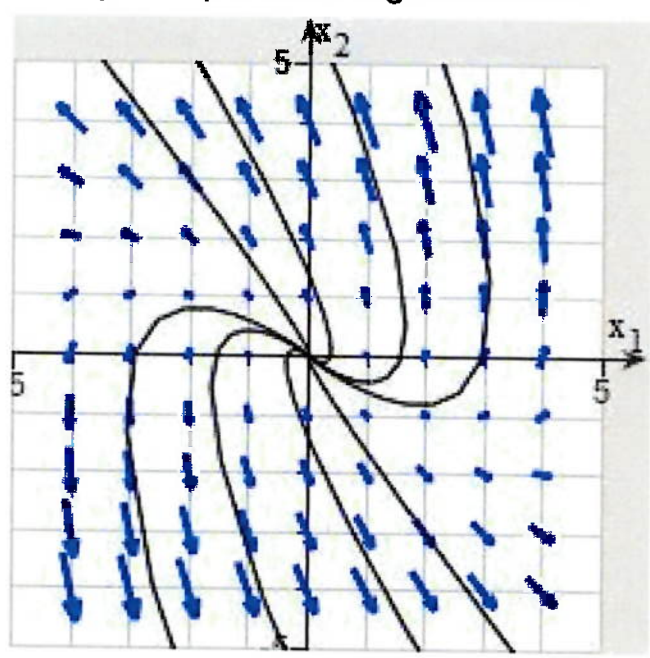


Proper Nodal Source: A repeated positive real eigenvalue with two linearly independent eigenvectors.



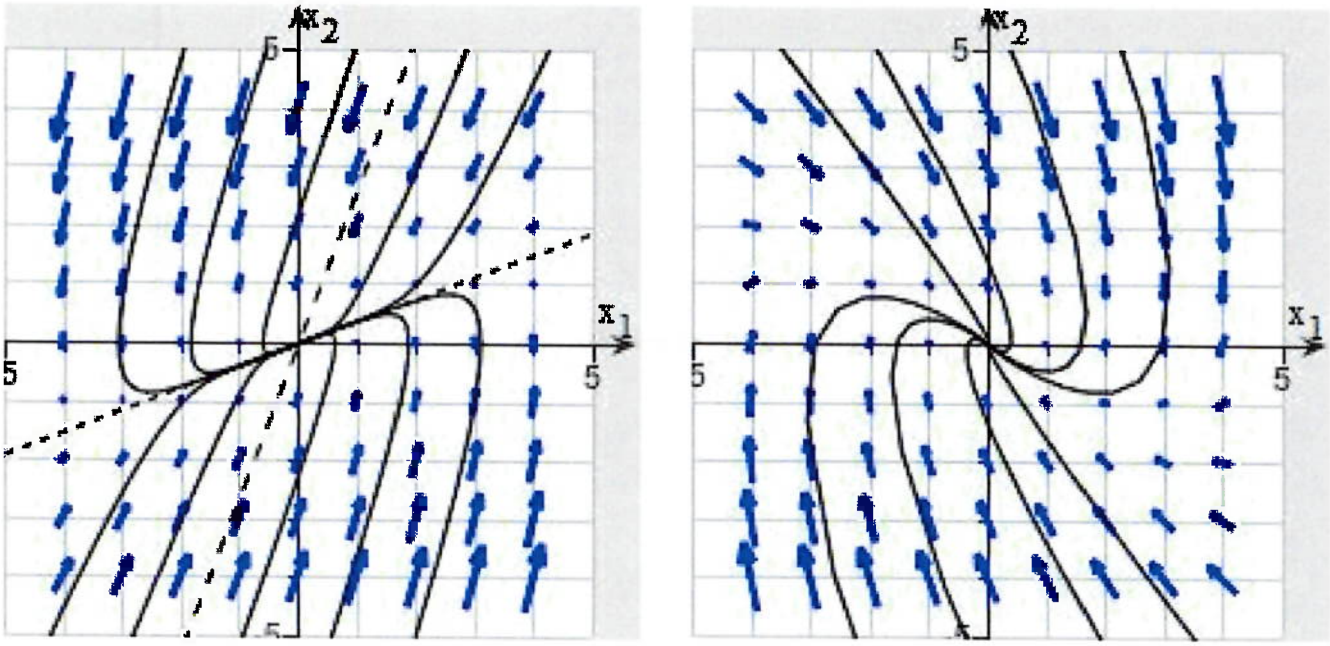
Proper Nodal Sink: A repeated negative real eigenvalue with two linearly independent eigenvectors.



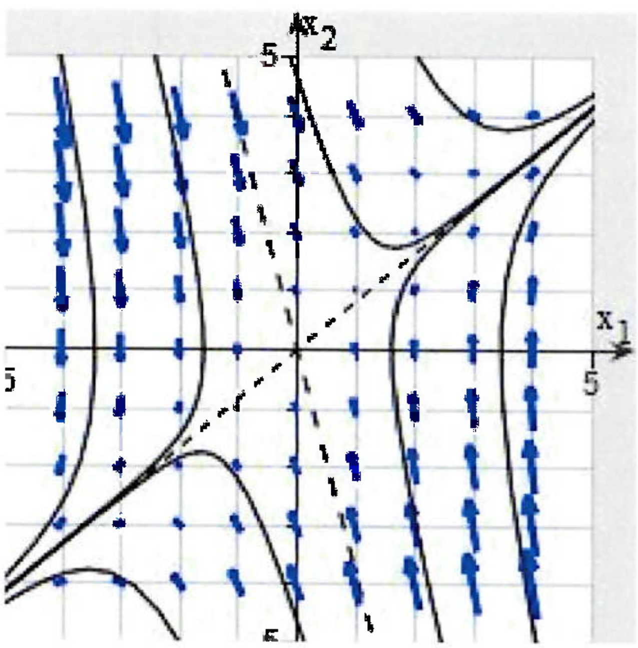


Improper Nodal Source: Distinct positive real eigenvalues (left) or a repeated positive real eigenvalue without two linearly independent eigenvectors (right).

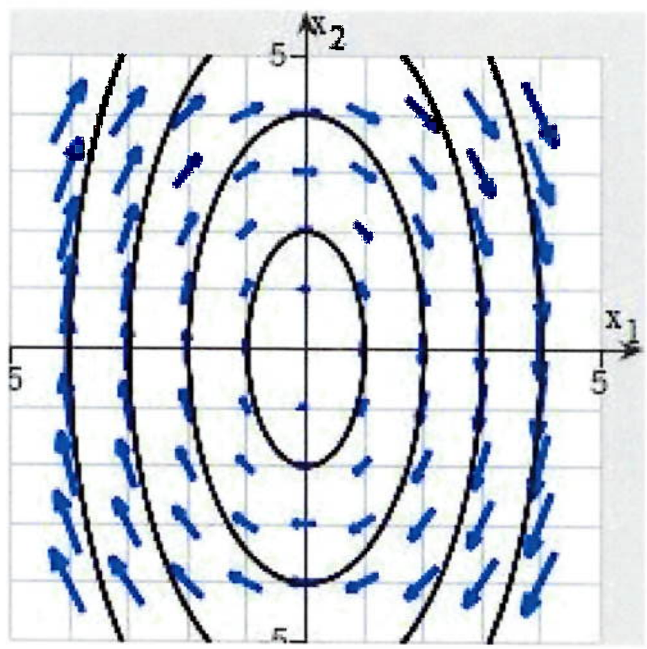
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Improper Nodal Sink: Distinct negative real eigenvalues (left) or a repeated negative real eigenvalue without two linearly independent eigenvectors (right).

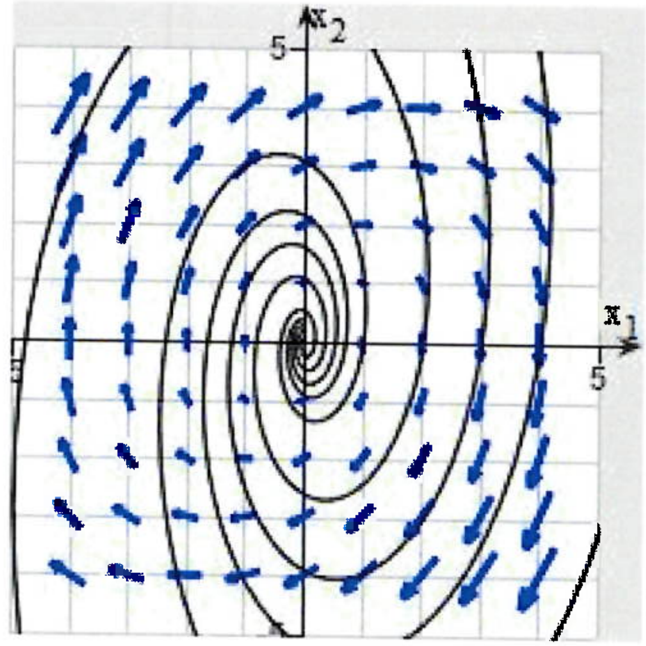


Saddle Point: Real eigenvalues of opposite sign.



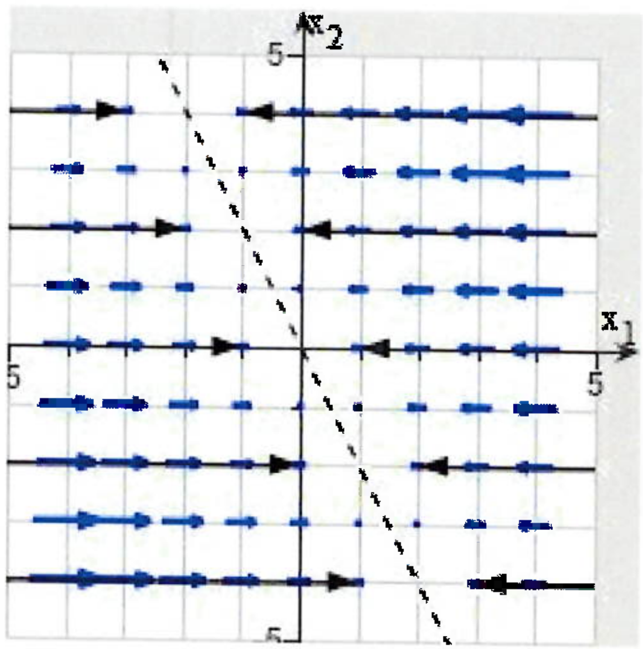
Center: Pure imaginary eigenvalues.

Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

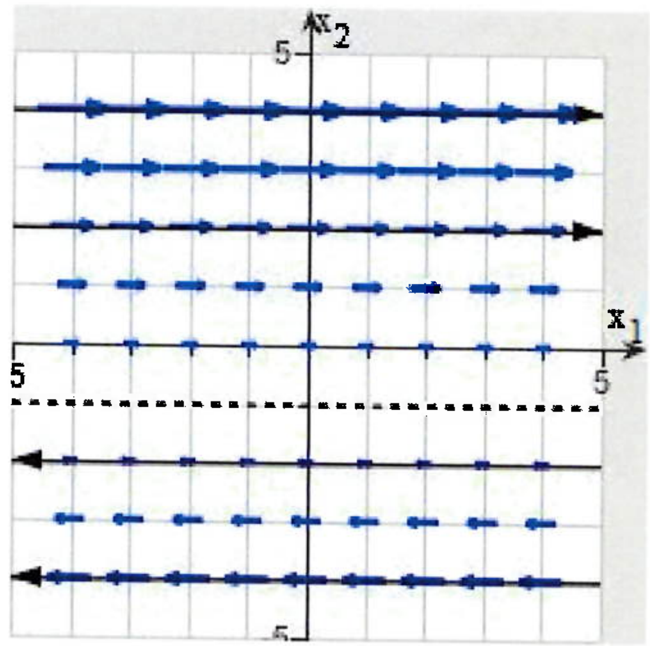


Spiral Source: Complex conjugate eigenvalues with positive real part.

Spiral Sink: Complex conjugate eigenvalues with negative real part.



Parallel Lines: One zero and one negative real eigenvalue. (If the nonzero eigenvalue is positive, then the trajectories flow away from the dotted line.)



Parallel Lines: A repeated zero eigenvalue without two linearly independent eigenvectors.